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A benchmark on sensitivity analysis tools applied to analytical test models

Scope of the benchmark within PAMINA

PAMINA, Task 2.1.D: Techniques for Sensitivity and Uncertainty Analysis

- Topic 1: Sensitivity Analysis benchmark using analytical test models

Benchmark participants:

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Objectives of the benchmark:

- Gaining experience with sensitivity analysis techniques
- Gaining experience with available software implementations
- Comparison of results
- Code debugging
- Conclusions on practical issues

Introduction: Sensitivity Analysis

Distinction from Uncertainty Analysis:

- What is the uncertainty in $Y(X_i)$ given the uncertainties in X_i ?
→ **Uncertainty Analysis**
- How important are the uncertainties of the individual inputs X_i with respect to the uncertainty in $Y(X_i)$? → **Sensitivity Analysis**

“important”
“relevant”
“sensitive”

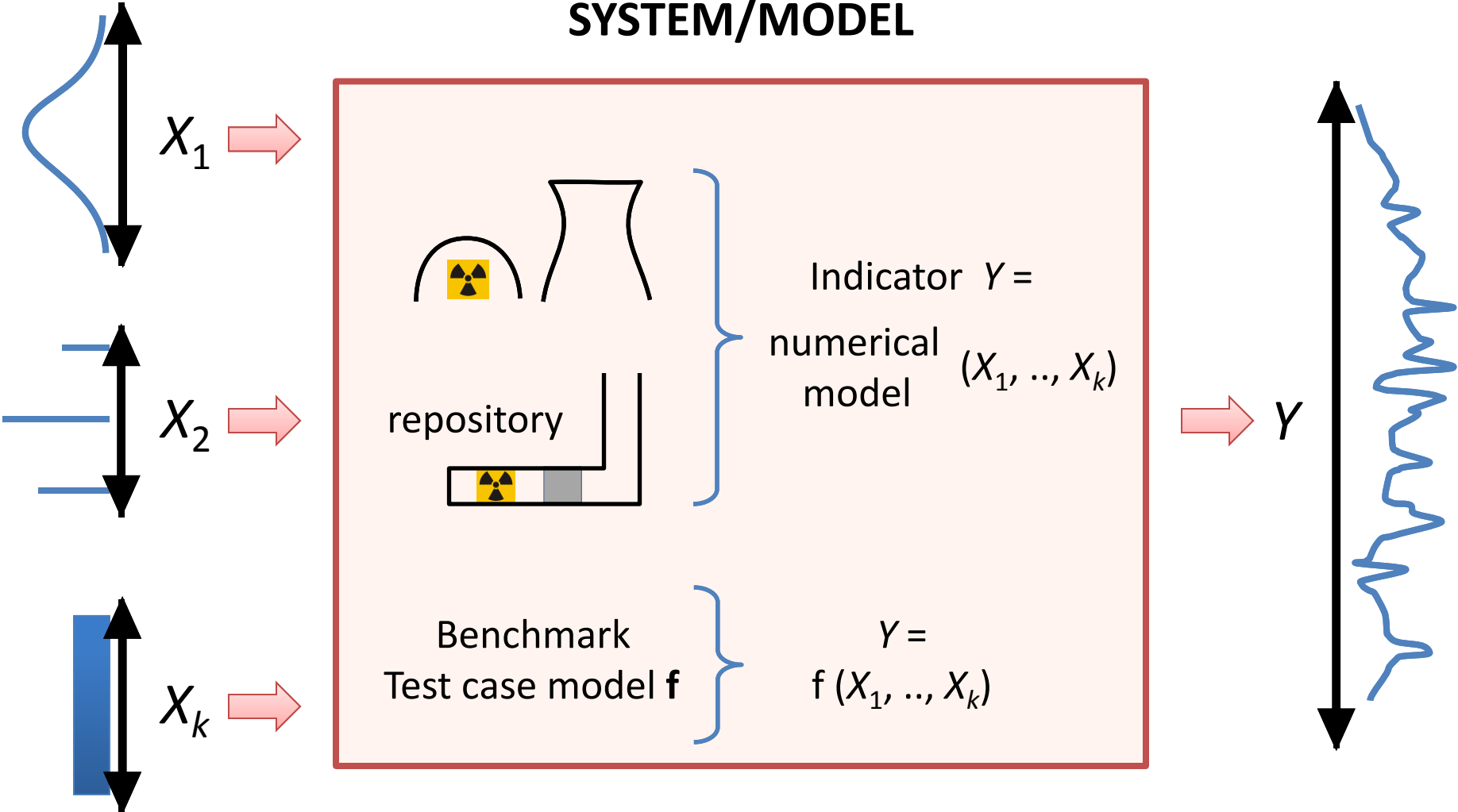
....

Interpretations differ !!!

Definitions: **Sensitivity Analysis ...**

- is a quantitative examination of how the behaviour of a system varies with change, usually in the values of the governing parameters (IAEA Safety Glossary)
- is the study of how the variation in the output of a model can be apportioned to different sources of variation, and of how the given model depends upon the information fed into it (A. Saltelli, 2000)
- studies the relationship between information flowing in and out of the model (A. Saltelli, 2000)

Introduction: Sensitivity Analysis



Introduction: Sensitivity Analysis

Two main purposes:

1. Improvement of system understanding
2. Identification of important/relevant/sensitive parameters

Beware of the many “methods” of Sensitivity Analysis (SA)!

Distinguish

- different purposes from
- different (sensitivity) measures from
- different probabilistic strategies and algorithms from
- different application cases/models

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In this benchmark:

- Purpose: Parameter importance ranking
- Measure: One fixed (“Main effect”)

Scope of the benchmark

Concentrate on one measure:

The ratio of the Variance of the Conditional Expectation to the (unconditional) Variance

$$S_i = \frac{\text{Var}[E[Y | X_i]]}{\text{Var}[Y]}$$

Properties:

- $0 \leq S_i \leq 1$
- X_i and Y independent $\rightarrow S_i = 0$
- $S_i = 1 \Leftrightarrow Y$ is function of X_i (only)

(also known as: Sensitivity indices, Correlation Ratios, Main Effects, Sobol' indices, ...)

Meaning:

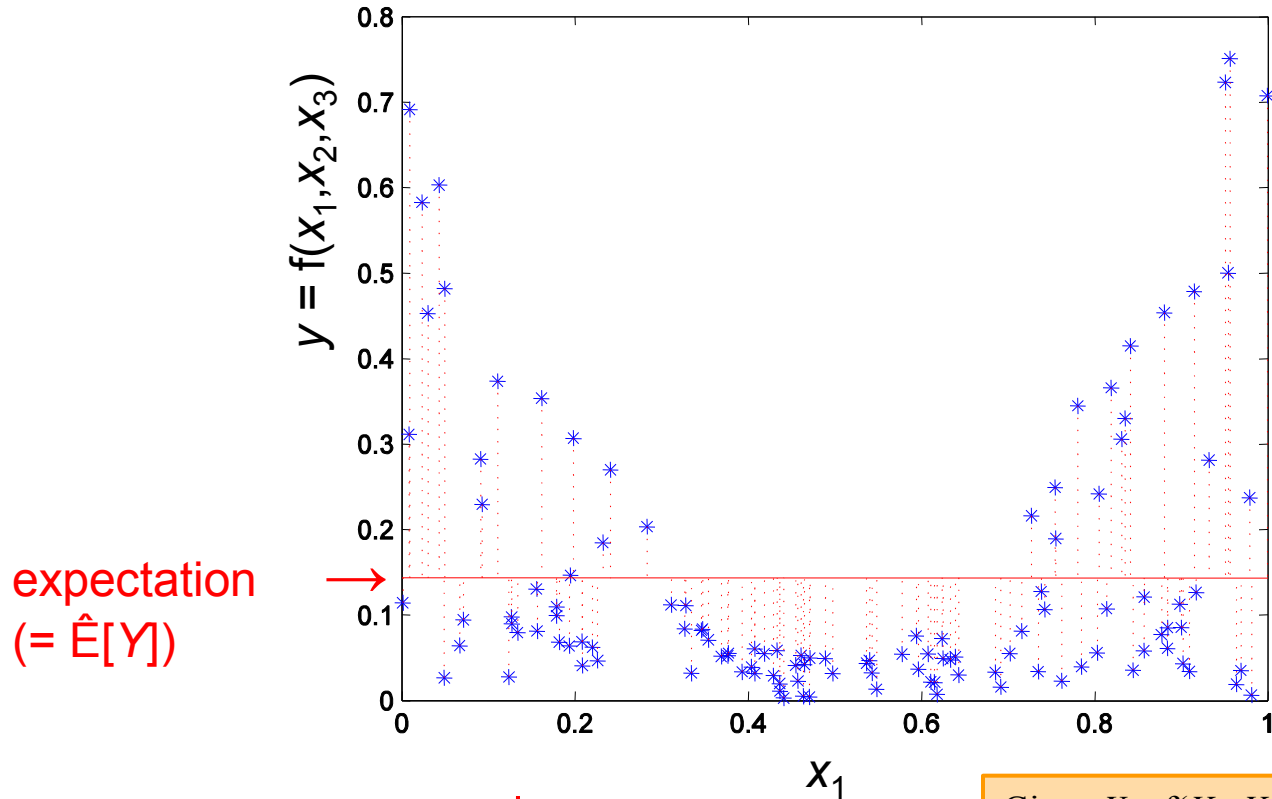
Fraction of variance of Y explained by a functional dependency on X_i

One measure — different strategies to estimate it

Used in benchmark: ECV, IHS, ... (“methods/algorithms/implementations”)

Illustration of estimating the variance

Scatter plot of X_1 for random sample of size 120

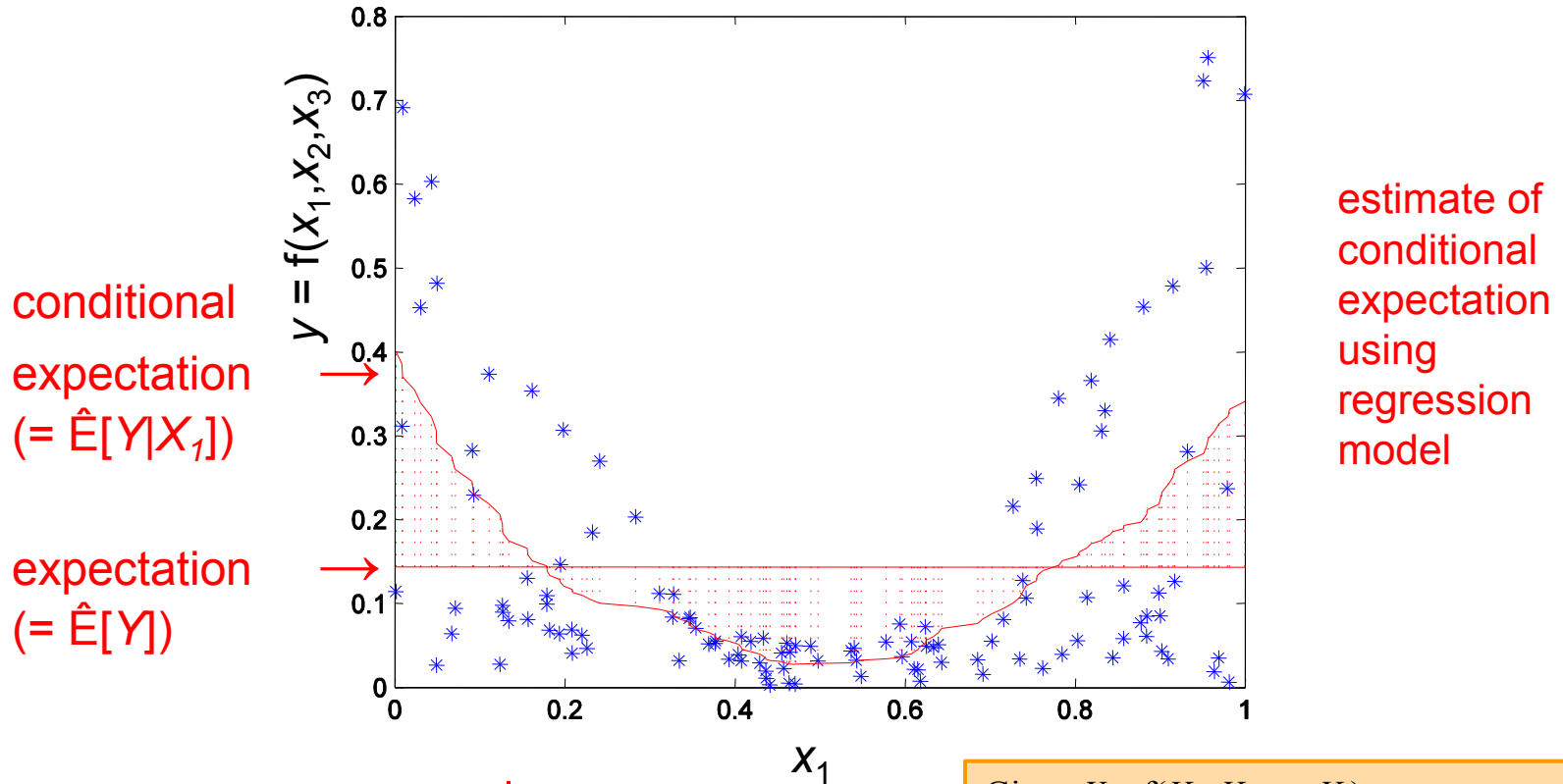


variance = sum of | squared

Given $Y = f(X_1, X_2, \dots, X_k)$:
For a considered input X_i , the **scatter plot** is the projection of the sample points of $(X_1, X_2, \dots, X_k, Y)$ on the (X_i, Y) plane.

Illustration of estimating the variance

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Scope of the benchmark

Participants use different software tools, hence

- different statistical strategies with different algorithms/implementations are employed (“methods”)

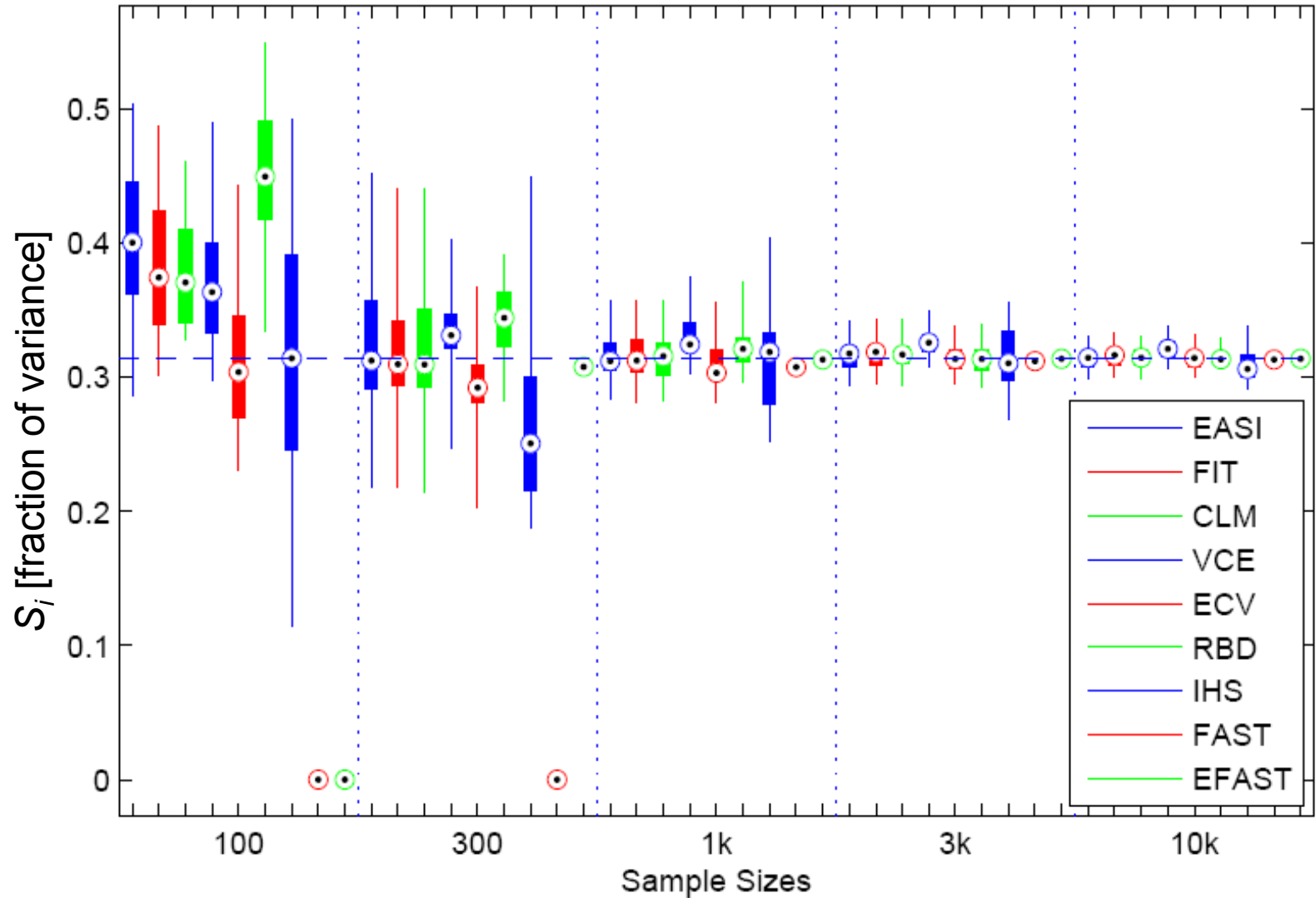
Four model functions are chosen with different peculiarities

- “Ishigami function” with 3 input parameters (non-monotonic)
- Switch model with 2 input parameters (discontinuous)
- Linear model with 2 dependent input parameters
- “Sobol’ g-function” with 8 input parameters (non-monotonic)

25 runs for each sample size (100, 300, 1000, 3000, 10000) performed

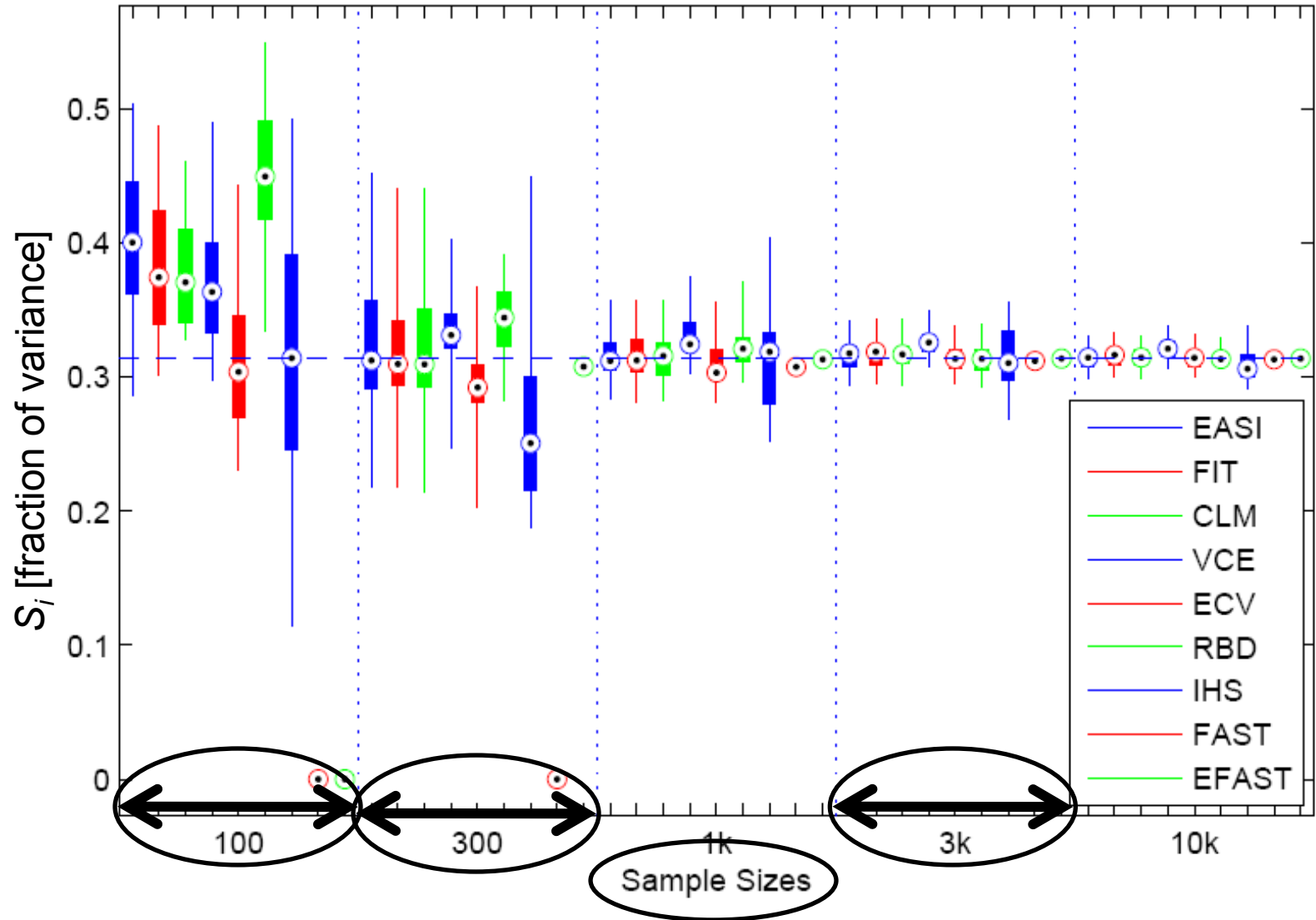
Some benchmark results

Comparison of methods. Here: Ishigami function, S_1



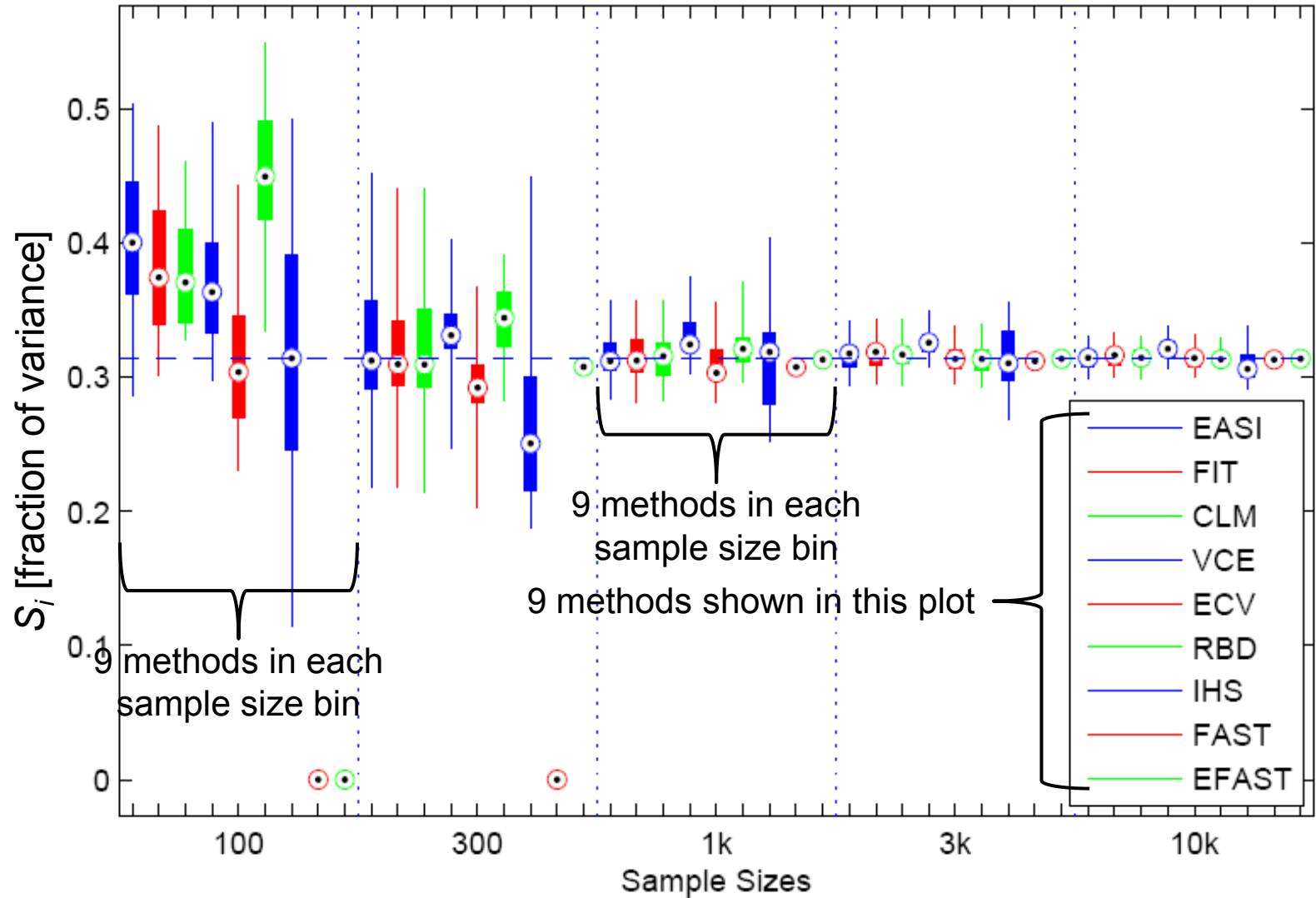
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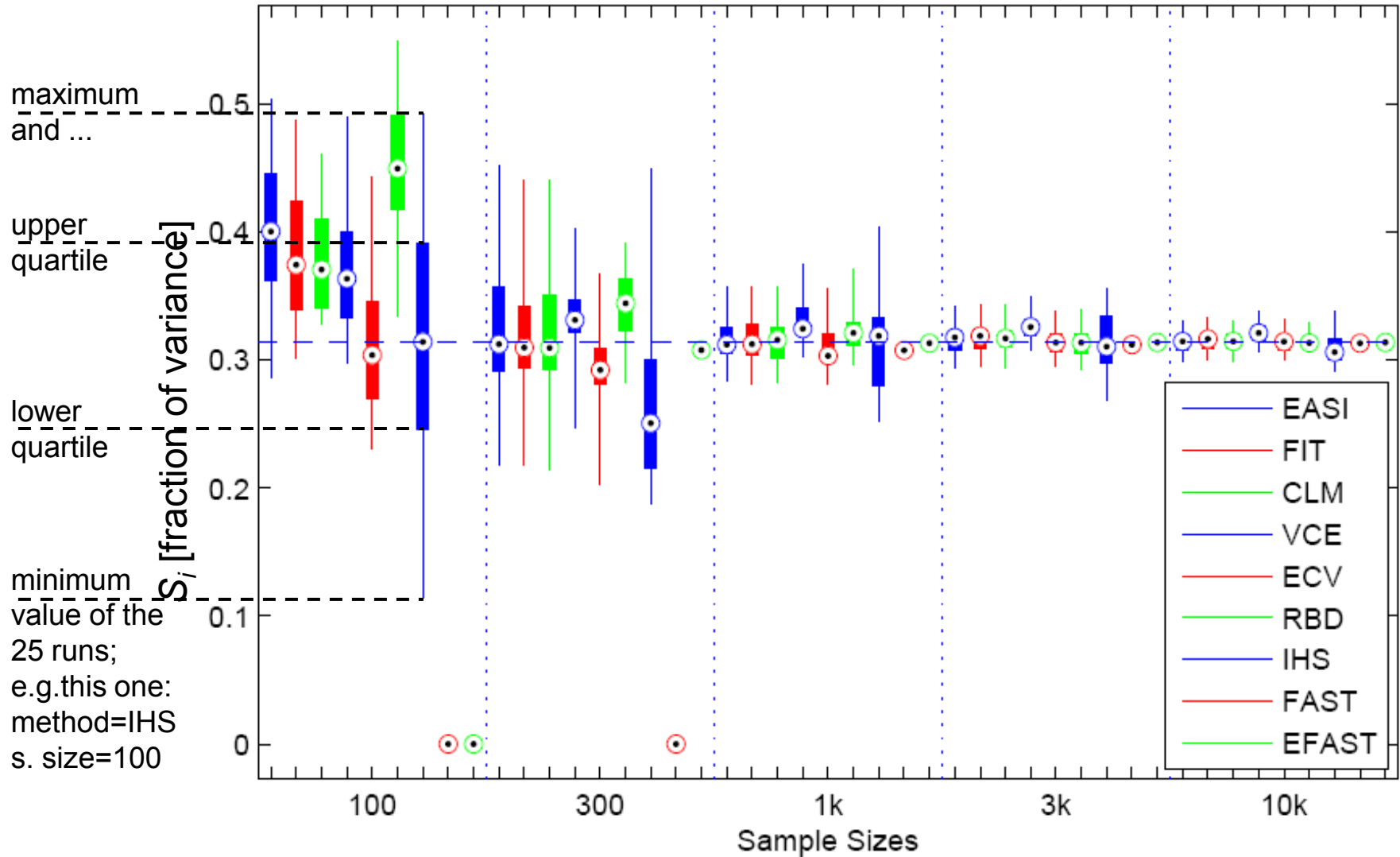
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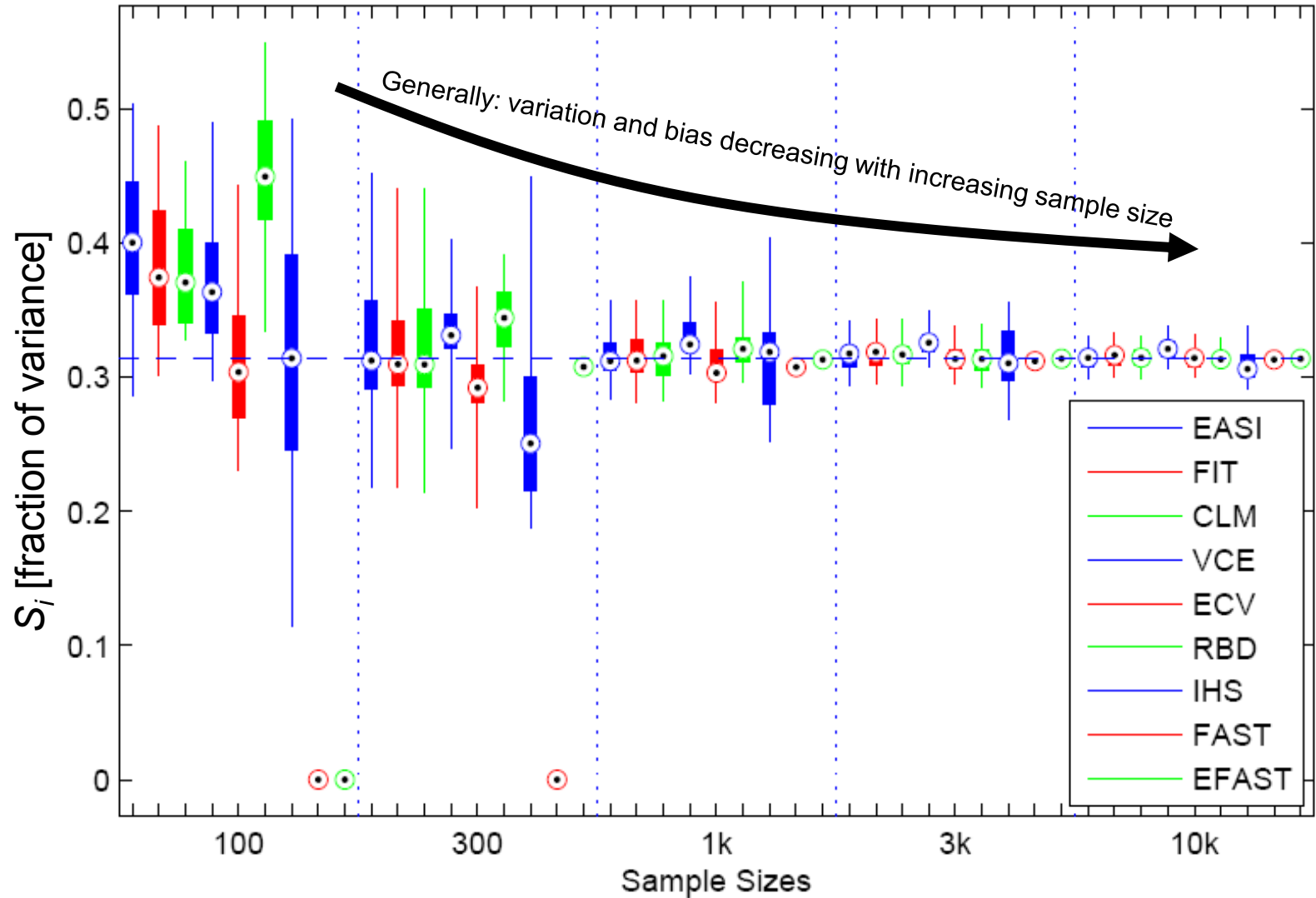
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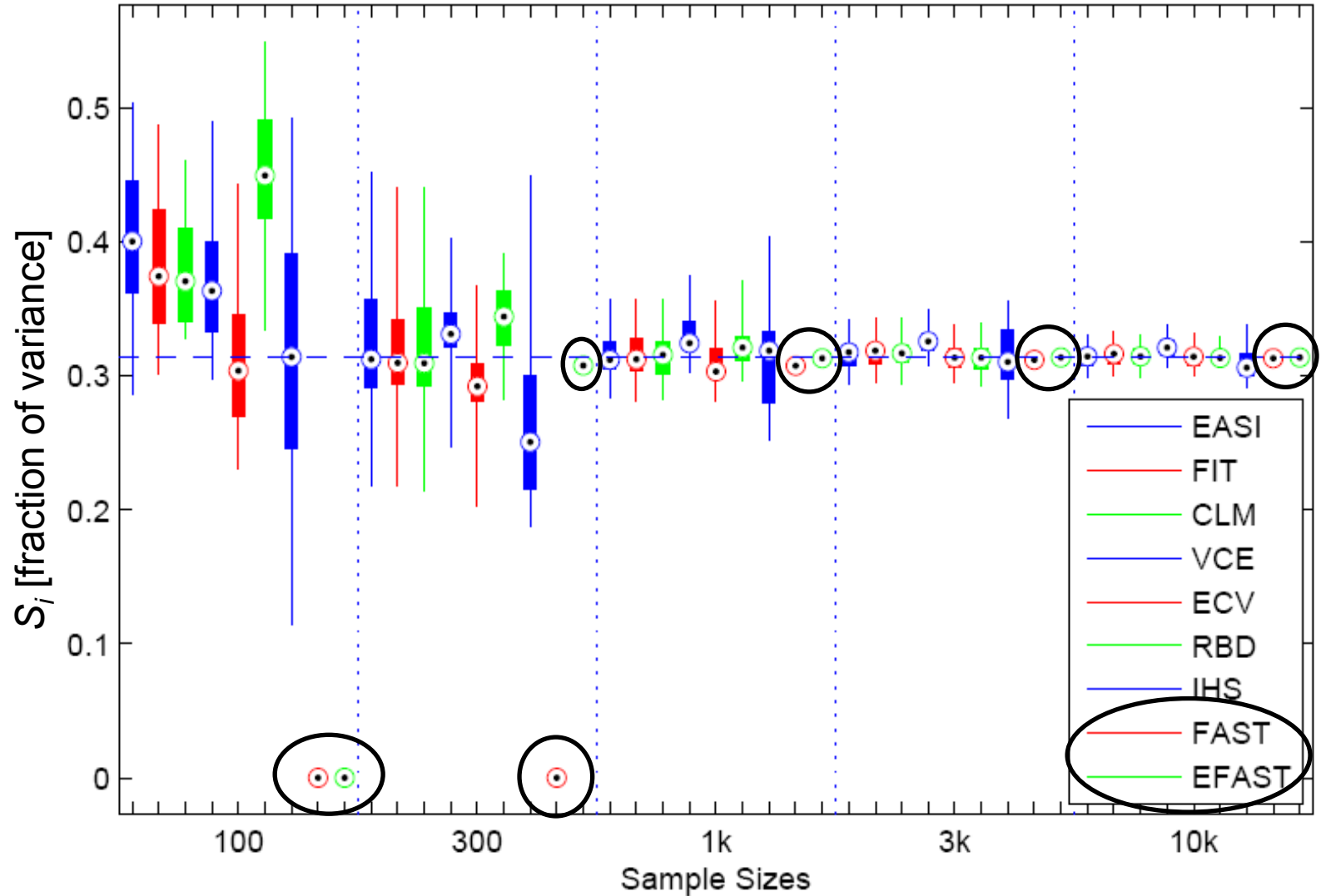
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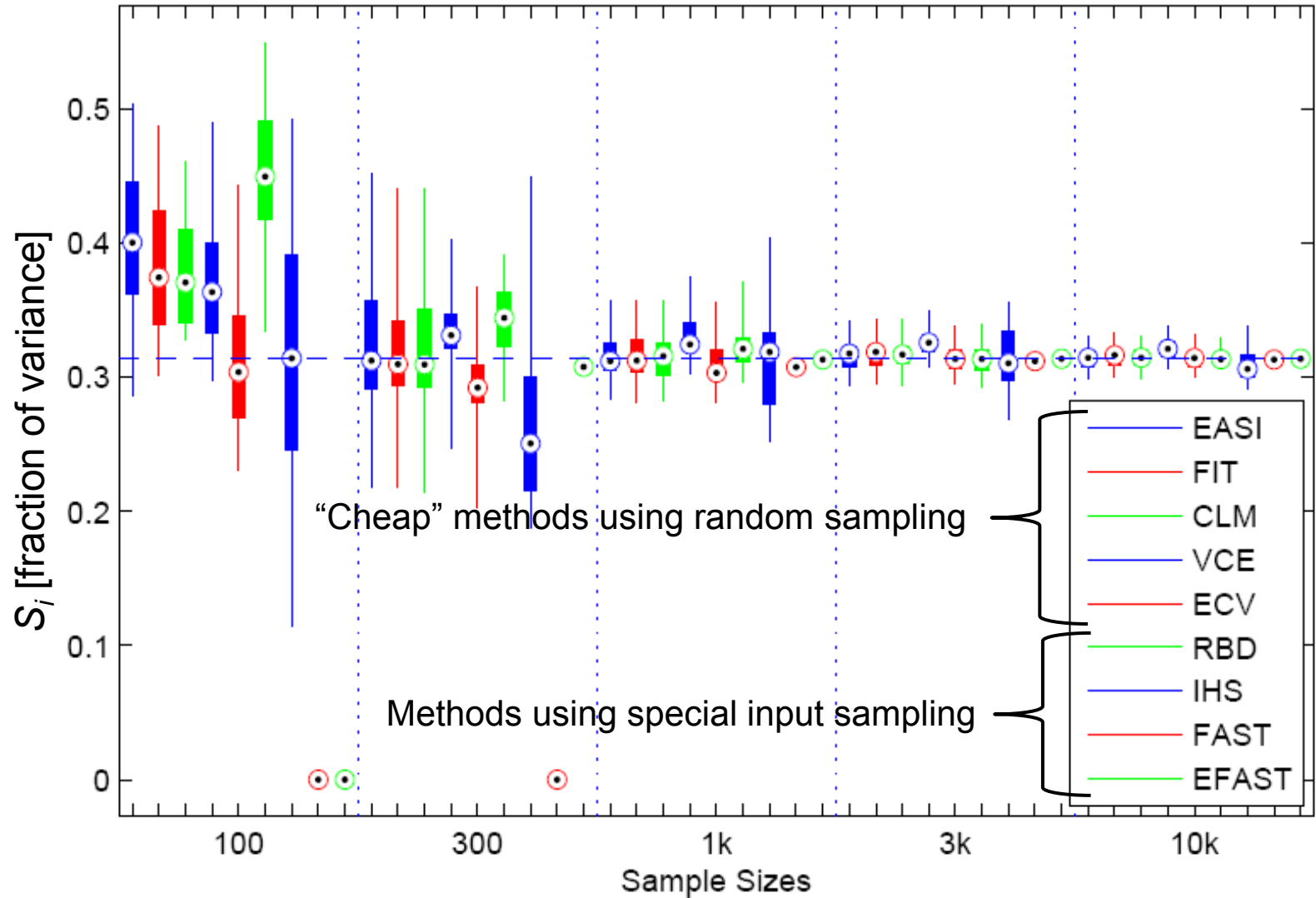
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Some benchmark results

Comparison of methods. Here: Ishigami function, S_1



Conclusions

- Benchmark setup proved very useful for comparison of algorithms
- Gained insight into the internals of variance-based sensitivity analysis
- Standard algorithms usually very stable, results with only minimal differences
- However, in certain cases, results depend substantially on the algorithm used
- No “best” method/algorithm available to be recommended in all cases
- All details of lessons learnt in milestone report

Outlook:

- Further research should lead to practical list of recommendations; based on standard cases
- General guideline for use of Sensitivity Analysis in Performance Assessment may be long-term goal

Thank you for your attention

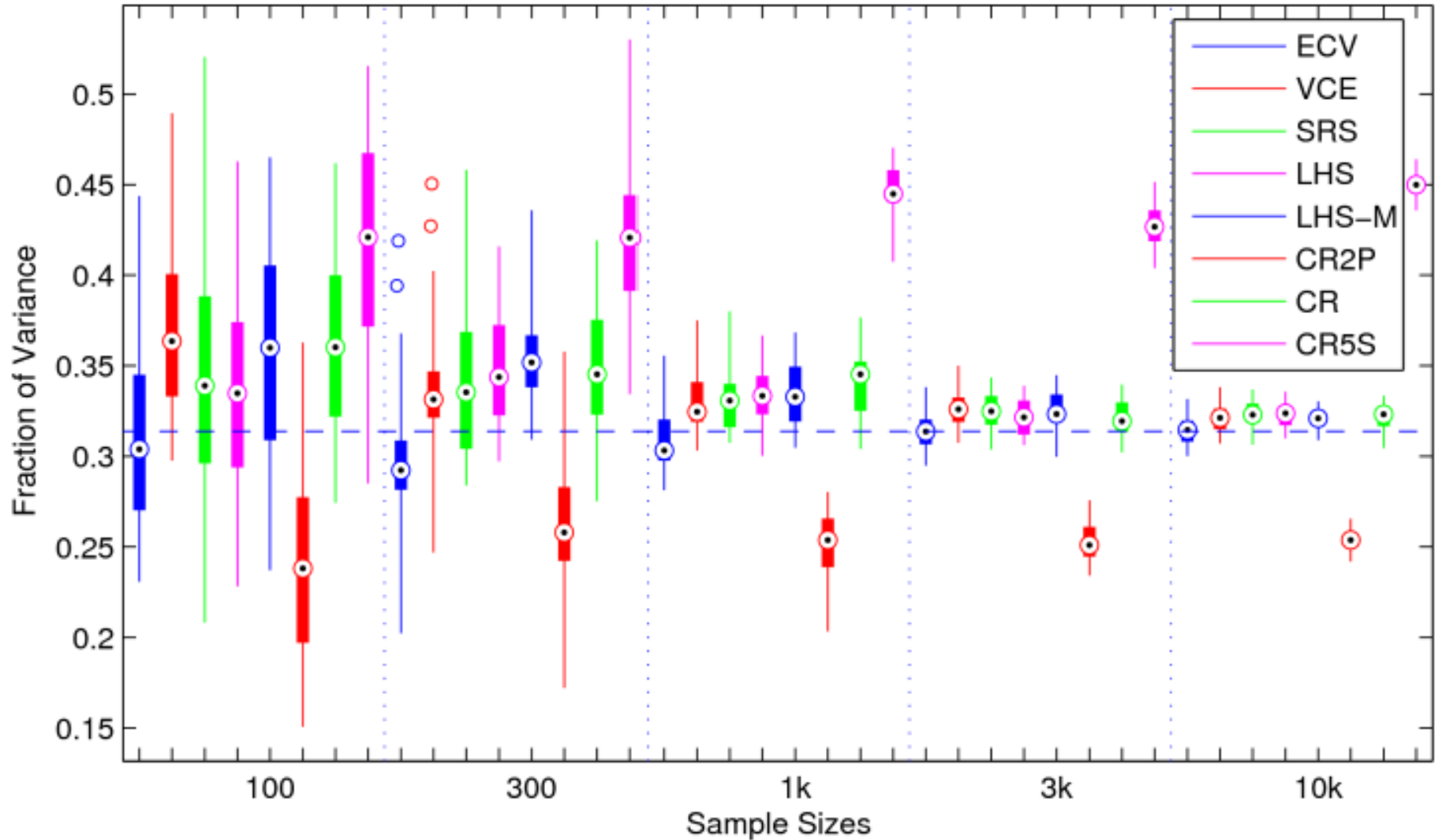
- All details on:
- <http://www.ip-pamina.eu/publications/reports/index.html>
→M2.1.D.11

More results ...

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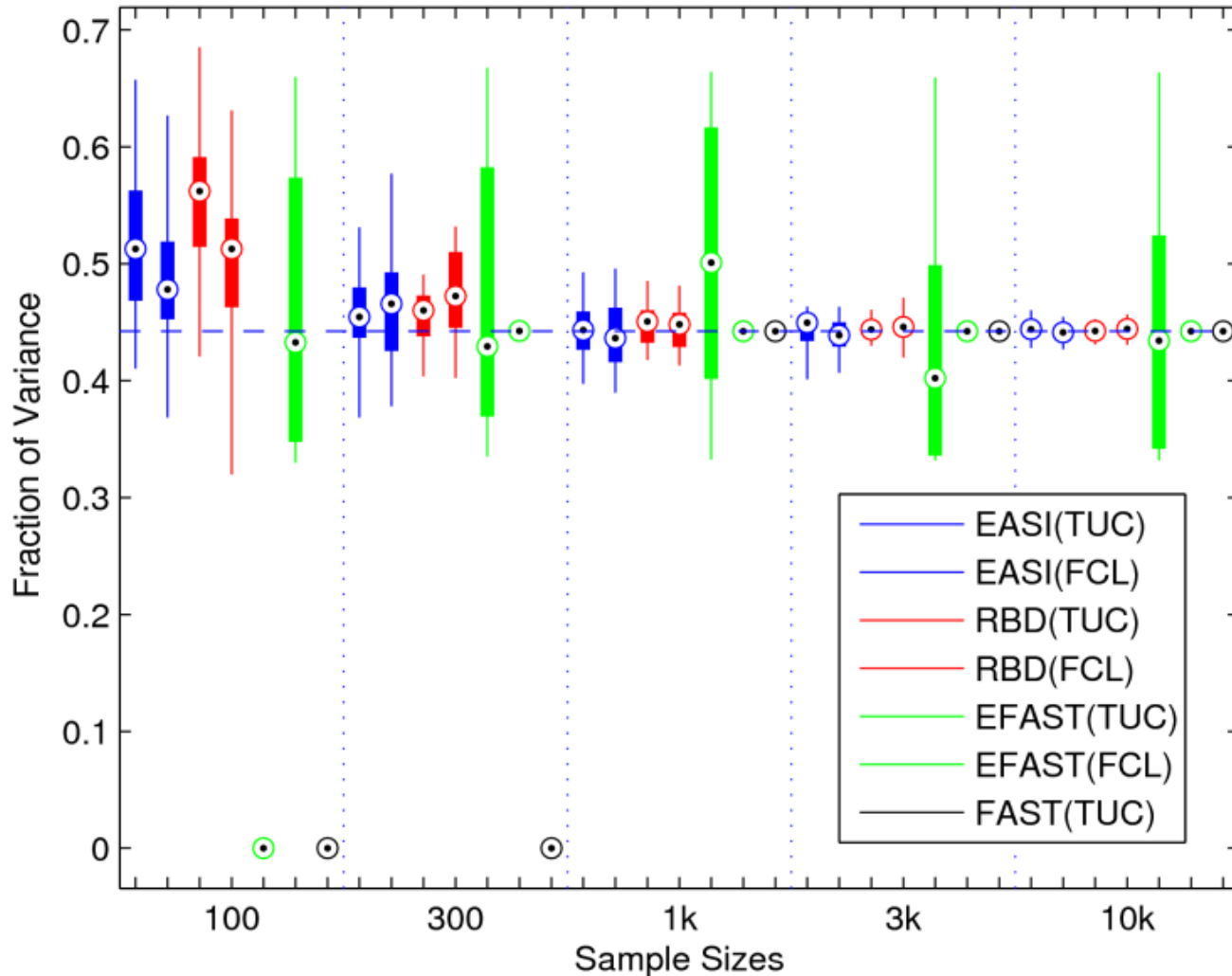
More benchmark results

Comparison of methods. Here: Ishigami function, S_1 , CR type methods only



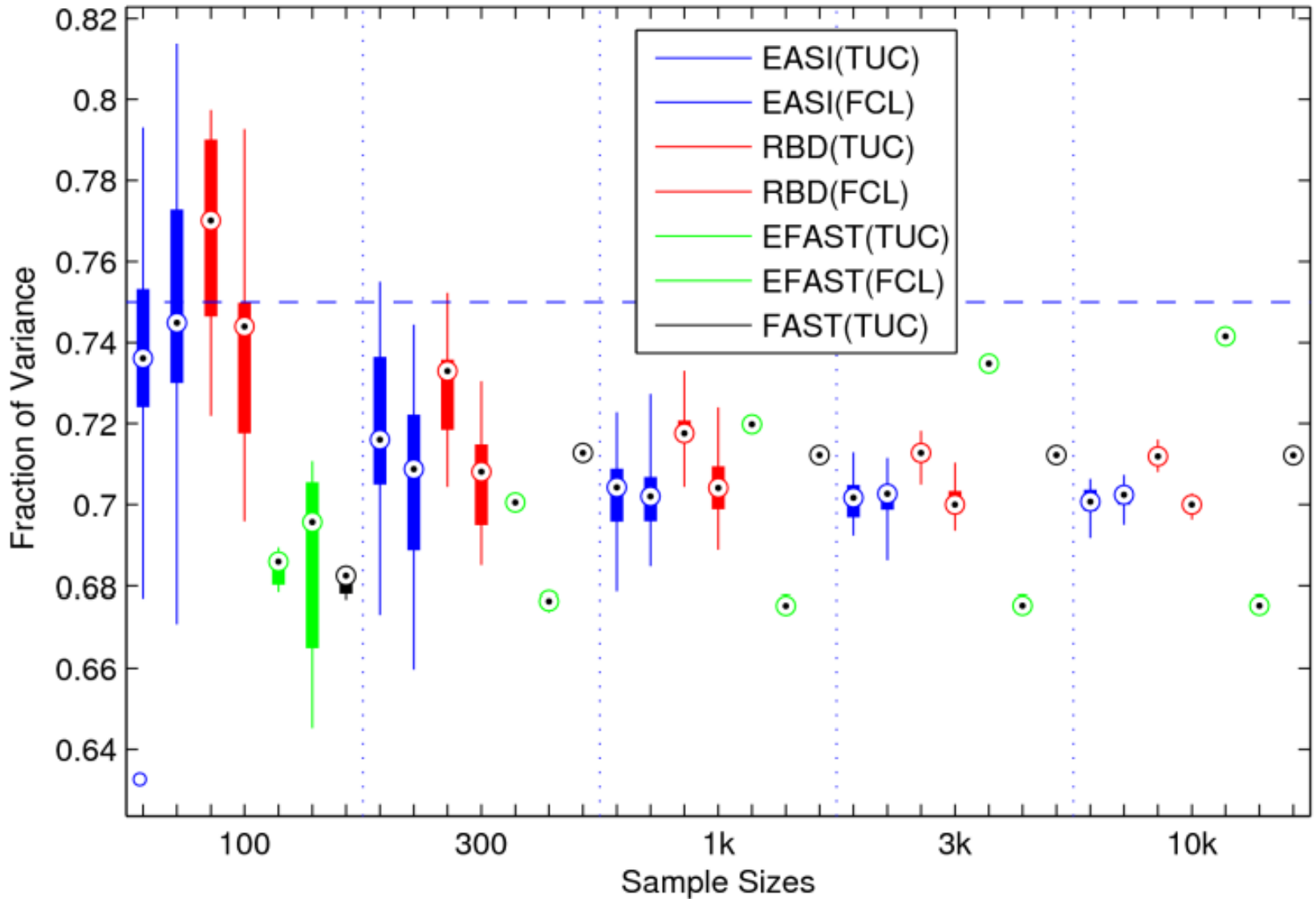
More benchmark results

Comparison of methods. Here: Ishigami function, S_2 , Fourier-based methods only



More benchmark results

Comparison of methods. Here: Switch model, S_1 , Fourier-based methods only



More benchmark results

Comparison of methods. Here: Linear dependent inputs model, S_2

